

# MIXED COLD-HOT DARK MATTER MODELS WITH SEVERAL MASSIVE NEUTRINO TYPES

Dmitri Pogosyan

*CITA, University of Toronto  
Toronto ONT M5S 1A7, Canada*

and

Alexei Starobinsky

*Landau Institute for Theoretical Physics  
Kosygina St. 2, Moscow 117334, Russia*

*Presented at the 11th Potsdam Workshop on Relativistic Astrophysics, September 1994  
To be published in *Astrophys. J.**

## ABSTRACT

Mixed cold-hot dark matter cosmological models (CHDM) with  $\Omega_{tot} = 1$ , approximately flat initial spectrum of adiabatic perturbations and 1, 2 or 3 types of massive neutrinos are compared and tested using recent observational data. The models with 2 or 3 neutrino types of equal mass permit as the best fit larger values of both the Hubble constant ( $H_0 \leq 60$  for 2 types,  $H_0 \leq 65$  for 3 types) and the total  $\Omega_\nu$  (up to 0.3 for 3 types) than the model with 1 massive type. Also, they have less problems with abundances of early compact objects including  $Ly - \alpha$  clouds.

## 1. Introduction

If the classification of different cosmological models by a number of additional fundamental parameters used in them to explain all observational data<sup>1,2</sup> is applied to inflationary models, then the model of the first level having only one fundamental parameter – an amplitude of perturbations – appears to be the CDM model with the approximately flat (Harrison-Zeldovich, or  $n \approx 1$ ) spectrum of initial adiabatic perturbations. Because of theoretical considerations and observational uncertainties, it is better to include “weakly-tilted” models with  $|n - 1| \leq 0.1$  into this class, too.

At present, it is clear already that predictions of this model, though being not far from observational data (that is remarkable for a such a simple model with only one free parameter), still definitely do not agree with all of them. Namely, if the free parameter is chosen to fit the data on scales  $(100 - 1000)h_{50}^{-1}$  Mpc, discrepancy of about twice in perturbation amplitude arises on scales  $(1 - 10)h_{50}^{-1}$  Mpc, and vice versa ( $h_{50} = H_0/50$ , where  $H_0$  is the Hubble constant in km/s/Mpc). Thus, models of the next (second) level having one more additional constant have to be considered. Among the best of such models is certainly the mixed cold+hot dark matter model (CHDM)<sup>3</sup>, for recent analysis see our previous papers (hereafter PS1<sup>4</sup> and PS2<sup>2</sup>), as well as<sup>5,6</sup> and references therein. In this model, the hot component is assumed to be the most massive of 3 neutrino species (presumably,  $\tau$ -neutrino) with the standard concentration following from the textbook Big Bang theory, while masses of the other

two types of neutrinos are supposed to be much less and, therefore, unimportant for cosmology. Then the only new fundamental parameter is the neutrino mass  $m_\nu$ .

Still the CHDM model with  $n \approx 1$  is not without difficulties. The main of them is connected with later galaxy and quasar formation in this model as compared to the SCDM model. As a result, only a small region in the  $H_0 - \Omega_\nu$  plane remains permitted (PS1<sup>4</sup>, some authors have a more pessimistic view<sup>7</sup>). The analysis of the possibility of non-flat (but power law) initial spectra (the article<sup>8</sup> for  $n < 1$ , PS2<sup>2</sup> for detailed analysis of  $n \geq 1$  case) leads to the conclusion that the CHDM works the best with nearly flat  $n \approx (0.95 - 0.97)$  initial spectra, which follow from the simplest inflationary models. Therefore, allowing for this degree of freedom does not improve the fit to observational data.

In this paper we consider how the cosmological predictions of the CHDM change if not only one but two or even all three types of neutrino are massive and contribute to the present density of hot matter. The total energy density (in terms of critical one) of hot component in this general case is then  $\sum m_{\nu_i} = 23.4\Omega_\nu h_{50}^2$  eV for  $T_\gamma = 2.73$ K.

This version of CHDM, of course, also corresponds to the next level of complexity in our classification until the mass ratio of different neutrino will be either confirmed in laboratory experiments, or theoretically derived from some underlying theory. We shall not cover all possible combination of the neutrino masses, noticing that the case of three equal masses  $m_{\nu_1} = m_{\nu_2} = m_{\nu_3}$  is the one mostly different from a standard model with one massive neutrino. The predictions for all other sets of  $m_{\nu_i}$  (with the same total  $\Omega_\nu$ ) lie “in between” these two models. There is no experimental evidence for all three types of neutrino to have comparable mass. Thus the model with  $m_{\nu_1} = m_{\nu_2} = m_{\nu_3}$  is mostly interesting as a limiting case. More realistic is the assumption of two type of neutrino to have mass in cosmologically interesting range while the rest one (electron neutrino) has a much smaller mass. Therefore we restrict ourselves with two models  $m_{\nu_1} = 0$ ,  $m_{\nu_2} = m_{\nu_3}$  and  $m_{\nu_1} = m_{\nu_2} = m_{\nu_3}$  and compare them with the original CHDM with one massive neutrino. We denote by  $N_\nu$  the number of (equally) massive types of neutrino which mass is now  $m_\nu = 23.4\Omega_\nu/N_\nu h_{50}^2$  eV.

## 2. CHDM Linear Perturbation Spectrum

The transfer function  $C(k)$  for the CHDM with one massive neutrino ( $N_\nu = 1$ ) was determined in our previous paper<sup>4</sup> numerically by solving a system of the Einstein-Vlasov equations for the evolution of adiabatic perturbations with a neutrino component treated kinetically and a CDM component as dust. The correspondent fitting formula is given in<sup>2</sup>. Results of similar computations for the case  $N_\nu = 2, 3$  are given in Fig.1 in comparison with  $N_\nu = 1$  curve for a specific choice  $\Omega_\nu = 0.2$ .

The main difference between models comes from the fact that the same total mass density fraction  $\Omega_\nu$  is achieved with different masses of neutrino. During the matter dominated stage the presence of hot component distributed at the beginning of this stage uniformly due to free-streaming lead to the retarded growth of perturbation in total density on modes  $kR_{nr} < 1$ . Here  $R_{nr}$  is the comoving size

Figure 1: Upper-left panel:  $C(k)$  transfer function for  $\Omega_\nu = 0.2$  and  $N_\nu = 1, 2, 3$  massive neutrino types. Other panels: Restrictions in the  $H_0 - \Omega_\nu$  parameter plane for  $N_\nu = 1$  (upper right),  $N_\nu = 2$  (lower right) and  $N_\nu = 3$  (lower left panel), following from: a) fit to Stromlo-APM counts-in-cells. Solid lines correspond to  $\chi^2 = 2, 7$  contours; b) the  $\sigma_8$  condition. The values  $\sigma_8 < 1, 0.67$  for  $Q_{rms-PS} = 14.3 \mu\text{K}$  are achieved left to the dashed (correspondingly rightmost and leftmost) lines. The middle dashed line corresponds to  $\sigma_8 = 1$  if  $Q_{rms-PS} = 19 \mu\text{K}$ ; c) the combination of the  $\sigma_8 < 0.67$  condition with quasar and galaxy formation conditions (dotted lines). The allowed region lies below the upper dotted line if the limitation set for objects of the mass  $M = 10^{11} \text{ M}_\odot$  is used and below the lower one if  $M = 10^{12} \text{ M}_\odot$ .

of horizon at the moment when neutrino become nonrelativistic. For the mode  $k$  this period continues until  $k$  becomes smaller than the effective neutrino Jeans scale  $k_J^2 = 4\pi G \rho_{tot}/(dp_\nu/d\rho_\nu) \propto m_\nu^2 t^{-2/3}$  when perturbation in neutrino density reach the magnitude of inhomogeneities in cold component (see discussion in PS1 <sup>4</sup>). This period is larger for smaller mass, and as the result, the amplitude of perturbation at the present moment is decreased even further from SCDM value. This explains why the transfer function  $C_{\text{CHDM}}$  is  $\approx 20\%$  lower for larger  $N_\nu$  at the intermediate scales  $R_{nr}^{-1} < k < k_J(t_0) \approx 11\Omega_\nu h_{50}^3 \text{ Mpc}^{-1}$ . For wavelength shorter than the present Jeans scale  $k > k_J(t_0)$  the perturbation in neutrino density did not start to grow up to the present moment and no additional dependence on  $m_\nu$  appear. In this limit, as Fig.1 demonstrates, the resulting amplitude is approximately the same as far as the total amount of neutrino  $\Omega_\nu$  is the same.

### 3. Confrontation with Observational Tests

In Fig.1 we present the restrictions in the  $\Omega_\nu - H_0$  parameter plane which follow from several observational tests shown to be the most illustrative in our previous papers. These tests are: a) value of the total *rms* mass fluctuation  $\sigma_8$  at  $R = 16h_{50}^{-1} \text{ Mpc}$  ( $\sigma_8 < 0.67$  based on cluster abundance data <sup>9</sup>) as follows from COBE measurement of DMR anisotropy ( $\Delta T/T$ ) which we adopt as  $Q_{rms-PS} = (17.4 \pm 3.1) \mu\text{K}$  <sup>13</sup>; b) The Stromlo-APM counts in cells <sup>10</sup> which limits the slope of spectrum over the range  $(20 - 150) h_{50}^{-1} \text{ Mpc}$ ; c) density of quasars at high redshifts  $z \approx 4$  <sup>11,12</sup>. Necessity to produce sufficient number of quasars in the model puts a lower limit on mass fluctuations at  $\propto 1 \text{ Mpc}$  scale. We follow <sup>12</sup> where the quasar density criterion at  $z = 4$  was formulated as a restriction on a mass fraction in bound objects with mass larger  $10^{11} \text{ M}_\odot$ :  $f(\geq 10^{11} \text{ M}_\odot) \geq 10^{-4}$ . Similar estimate <sup>12</sup> for a fraction of mass in large galaxies gives  $f(\geq 10^{12} \text{ M}_\odot) \geq 10^{-5}$  at the same redshift  $z = 4$ . We put greater weight on this second, more restrictive limit, because, remarkably, it gives similar restrictions on the CHDM model as the analysis of the abundance of damped  $Ly-\alpha$  absorption systems in high-redshift quasar spectra <sup>14,15</sup>.

The upper-right panel of Fig.1 presents the  $\Omega_\nu - H_0$  parameter plane with observational restrictions for standard CHDM with  $n = 0.95$ . This plot was discussed in detail both in PS1 <sup>4</sup> and PS2 <sup>2</sup>, showing that CHDM model parameters are restricted to the narrow range of a low Hubble constant  $H_0 \leq 55 \text{ km/s/Mpc}$  and the neutrino

fraction  $\Omega_\nu = 0.17 - 0.28$  for  $H_0 = 50$ . The first of these limits reflects a problem with unavoidable high mass fluctuations at the  $16h_{50}^{-1}$  Mpc scale (note that to set this limit we use a *lower* bound on  $(\Delta T/T)$ ,  $Q_{rms-PS} = 14.3 \mu\text{K}$  although recent analysis suggest higher value  $Q_{rms-PS} \approx 20 \mu\text{K}$ ), as well as a wrong shape of the perturbation spectrum over the  $l = (20 - 150)h_{50}^{-1}$  Mpc interval if  $H_0$  is high. The upper bound on  $\Omega_\nu$  comes from the combined quasar and  $\sigma_8$  conditions that implies that the slope of the spectrum in the scale range  $(0.7h_{50}^{-2/3} - 16h_{50}^{-1})$  Mpc cannot be too steep.

The bottom panels of Fig.1 correspond to two- and three- massive neutrino models  $N_\nu = 2, 3$ . The additional decrease of the transfer function in the range  $0.01\text{Mpc}^{-1} < k < 1\text{Mpc}^{-1}$  while keeping both larger and smaller scales unchanged allow us to noticeably relax the restrictions on the CHDM models. The major change is the possibility to accommodate the higher values of Hubble constant. Now both the shape of spectrum better fits count-in-cells and the  $\sigma_8$  limit, understandably, becomes less severe. For an extreme example  $N_\nu = 3$  we may have  $H_0$  as high as  $70 - 75 \text{ km/s/Mpc}$ . For  $N_\nu = 2$  the tests can be satisfied if  $H_0 \leq 65 \text{ km/s/Mpc}$ , although more comfortable fit to data is achieved for  $H_0 \leq 60$  ( $N_\nu = 2$ ) and  $H_0 \leq 65$  ( $N_\nu = 3$ ). Let us stress, however, that CHDM by itself does not address the problem of the age of the Universe, which remains the major theoretical objection to  $H_0 \geq 65 \text{ km/s/Mpc}$ .

Additionally a somewhat higher values of  $\Omega_\nu$  are now allowed. Although for  $H_0 = 50$  the boundaries on total neutrino fraction remain unchanged  $\Omega_\nu = 0.15 - 0.25$ , for  $H = 60$  and  $N_\nu = 2, 3$  we have from Fig.1  $0.2 < \Omega_\nu < 0.3$ . This change is rather significant as a limit on the sum of neutrino masses:  $3.5\text{eV} < \sum m_{\nu_i} < 6\text{eV}$  if we take  $H_0 = 50$  and  $6.7\text{eV} < \sum m_{\nu_i} < 10\text{eV}$  for  $H_0 = 60$ . On the other hand, the mass of a single neutrino type is approximately the same  $m_{\nu_i} \simeq 2 - 5 \text{ eV}$ .

We can conclude that having several types of neutrino with cosmologically significant mass of few electron-volts expands the boundary of parameter space for CHDM model. The allowed region of  $\Omega_\nu - H_0$  plane now include not only the small area of  $H_0 \approx 50$ ,  $\Omega_\nu \approx 0.2$  but extends to  $H_0 \approx 60$ ,  $\Omega_\nu \approx 0.25$  for the most physically interesting version with two massive neutrino, and as far as  $H_0 \approx 70$ ,  $\Omega_\nu \approx 0.3$  for the extreme case of three equally massive types. Similar results for the  $N_\nu = 2$  case were recently presented in <sup>16</sup>.

A.S. was supported in part by the Russian Foundation for Basic Research, Project Code 93-02-3631, and by Russian Research Project "Cosmomicrphysics". D.P. is grateful for the Organizing Committee of the 11th Potsdam Workshop on Relativistic Astrophysics for financial support.

1. A. Starobinsky, in *Theoretical and Experimental Problems of Gravitation* (Abstr. 8th Russian Grav. Conf., Pushchino, Moscow, 1993), p. 149.
2. D. Pogosyan and A. Starobinsky, *Astrophys. J.* **447** in press; (PS2).
3. Q. Shafi and F. W. Stecker, *Phys. Rev. Lett.* **53** 1292.
4. D. Pogosyan and A. Starobinsky, *Mon. N. R. Astr. Soc.* **265** 507; (PS1).
5. A. Liddle and D. Lyth, *Mon. N. R. Astr. Soc.* **265** 379.
6. A. Klypin, J. Holtzman, J. Primack and E. Regös, *Astrophys. J.* **416** 1.
7. R. Cen and J. P. Ostriker, *Astrophys. J.* **431** 451.

8. D. Lyth and A. Liddle, *preprint astro-ph/9401014*.
9. S. White, G. Efstathiou and C. Frenk, *Mon. N. R. Astr. Soc.* **262** 1023.
10. J. Loveday, G. Efstathiou, B. Peterson and S. Maddox, *Astrophys. J.* **400** L43.
11. G. Efstathiou and M. J. Rees, *Mon. N. R. Astr. Soc.* **230** 5P.
12. M. Haehnelt, *Mon. N. R. Astr. Soc.* **265** 727.
13. E. L. Wright, G. F. Smoot, A. Kogut et al., *Astrophys. J.* **420** 1.
14. A. Klypin, S. Borgani, J. Holtzman and J. Primack, *Astrophys. J.* in press.
15. C. Ma and E. Bertschinger, *preprint astro-ph/9407085*, *Astrophys. J.* in press.
16. J. Primack, J. Holtzman, A. Klypin and D. Caldwell, *preprint SCIPP 94/28, astro-ph/9411020*.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/astro-ph/9502019v2>